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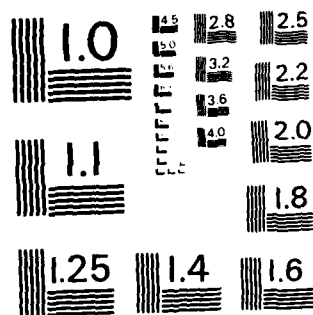
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) During the period 1 Jul 82 - 30 Jun 83, the principal investigator and the co-investigator attended four conferences, presenting papers at two of them. The principal investigator organized a conference on "Stochastic Failure Models, Replacement and Maintenance Policies, Accelerated Life Testing." He continued his research on shock models, wear processes, replacement and maintenance policies; revised the paper, "Life Distribution Properties of Devices Subject to a Levy Wear Process" that will appear in <u>Mathematics of Operations Research</u> , wrote the paper, "Pure Jump Damage Processes and submitted it for (CONT.)		

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ITEM #20, CONT.: publication, and finished the paper, "Pure Jump Wear Processes: A Review," which will appear in The Proceedings of the First Saudi Conference on Statistics and its Applications. He also was a key-note speaker at the First Saudi Conference on Statistics. The paper, "Conservative and Dissipative Parts of Non-Measure Preserving Weighted Composition Operators," by the co-investigator, appeared in the Houston Journal of Mathematics. The co-investigator presented a paper on "Approximate Optimal Replacement Policies and Their Stability," at the conference "Stochastic Failure Models, Replacement and Maintenance Policies, Accelerated Life Testing" that was organized by the principal investigator. The details of the research conducted and conferences attended are contained within the interim report.



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FINAL REPORT  
GRANT AFOSR 80-0245

From July 1, 1982 to June 30, 1983 /

During the period July 1, 1982 to June 30, 1983 the Principal Investigator and the Co-investigator attended four conferences giving papers at two of them. The Principal Investigator organized a Conference on "Stochastic Failure Models, Replacement and Maintenance Policies, Accelerated Life Testing". He continued his research on shock models, wear processes, replacement and maintenance policies; revised the paper "Life Distribution Properties of Devices Subject to a Lévy Wear Process", that will appear in Mathematics of Operations Research, wrote the paper "Pure Jump Damage Processes", and submitted it for publication, and finished the paper "Pure Jump Wear Processes: A Review" which will appear in The Proceedings of the First Saudi Conference on Statistics and its Applications. He also was a key-note speaker at the First Saudi Conference on Statistics. The paper, "Conservative and Dissipative Parts of Non-Measure Preserving Weighted Composition Operators", by the Co-investigator, appeared in the Houston Journal of Mathematics. The Co-investigator presented a paper on "Approximate Optimal Replacement Policies and Their Stability", at the conference "Stochastic Failure Models, Replacement and Maintenance Policies, Accelerated Life Testing" that was organized by the Principal Investigator. The details of the research conducted and conferences attended are as follows:

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# I. RESEARCH CONDUCTED

A) PURE JUMP DAMAGE PROCESSES. Assume that a device is subject to damage, denoted by  $X$ . The damages process is assumed to be an increasing pure jump process. Such a process can be expressed as follows:

$$X_t = X_0 + X_t^d,$$

where

$$X_t^d = \sum_{u \leq t} (X_u - X_{u-}).$$

Clearly  $X_t^d$  describes the cumulative wear due to shocks occurring during  $(0, t]$  and  $X_0$  is the initial wear.

Let  $(t, K)$  be the Lévy system for  $X$ . It is known from the work of E. Benveniste and J. Jacod that, for each positive Borel measurable function  $f$  on  $R_+ \times R_+$  with  $f(x, x) = 0$ , for all  $x$  in  $E$ .

$$\begin{aligned} E_x \left[ \sum_{s \leq t} f(X_{s-}, X_s) \right] \\ = E_x \left[ \int_0^t ds \int_{R_+} K(X_{s-}, dy) f(X_{s-}, y) \right] \end{aligned}$$

Cinlar and Jacod show that there exists a Poisson random measure on  $R_+ \times R_+$  whose mean measure element at  $(s, z)$  is  $du dz/z^2$  and a deterministic function  $c$  such that

$$\sum_{u \leq t} f(X_{s-}, X_s) = \int_{[0, t] \times R_+} N(ds, dz) f(X_{s-}, X_{s-} + c(X_{s-}, z))$$

almost everywhere for each function  $f$  on  $R_+ \times R_+$  of the above type.

In particular, it follows that

$$X_t = X_0 + \int_{[0, t] \times R_+} c(X_{s-}, z) N(ds, dz)$$

The above formula has the following interpretation:

$t \rightarrow X_t(w)$  jumps at  $s$  if and only if the Poisson random measure  $N(w, \cdot)$

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has an atom  $(s, z)$  and then the jump is from the left hand limit  $X_{s-}(w)$  to the right hand limit

$$X_s = X_{s-} + c(X_{s-}, z) .$$

The function  $c(x, z)$  represents the damage due to a shock of magnitude  $z$  occurring at a time when the previous cumulative wear equals  $x$ .

We determined conditions on the function  $c$  and the Lévy kernel  $K$  that insure that the life distribution properties of the threshold right tail probability  $\bar{G}$  are inherited as corresponding properties of the survival probability  $\bar{F}(t) = E[G(X_t)]$ . We note that a stationary Lévy shock and wear process is a special case of the pure jump processes described above. This follows from Ito's representation of every increasing Lévy process as follows:

$$X_t = X_0 + \int_{[0, t] \times R_+} z N(ds, dz) ,$$

where  $N$  is a Poisson random measure with mean measure element at  $(s, dz)$  equals  $ds \mu(dz)$  and  $\mu$  is a Lévy measure.

These results have been submitted for publication.

#### B) OPTIMAL REPLACEMENT AND MAINTENANCE POLICIES FOR DEVICES SUBJECT TO PURE JUMP DAMAGE PROCESSES.

Assume that a device is subject to damage occurring randomly in time according to a pure jump process  $X$ . Let  $g(w, t): \Omega \times R_+ \rightarrow R$  be a function describing the cumulative net yield the device produces till time  $t$ . Suppose that  $g(w, t) = f(X_t(w))$  for some  $f: R_+ \rightarrow R$ . The expected cumulative discounted net yield from observing the device till time  $t$  equals  $E[d(t)f(X_t)]$ . We give conditions on the functions  $d$  and  $f$  that ensures the existence of a replacement policy that maximizes the expected cumulative discounted net yield and give a closed form for that replacement time. In the case  $d(t) = e^{-\lambda t}$  we show that for a suitable choice of the function  $f$  the optimal

replacement policy is a control policy.

The following theorem follows from the work of Ikeda and Watanabe or Benveniste and Jacod and the martingale characterization theorem for Poisson random measures due to Jacod. The theorem holds for a larger class of functions than the one indicated. However, the statement given here is sufficient for our purpose. First define

$$Af(x) = \int_{R_+ \setminus \{0\}} [f(x+c(x,z)) - f(x)] \frac{dz}{z^2}.$$

Theorem. Let  $f: R_+ \rightarrow R$  be increasing  $d: R_+ \rightarrow R_+$  be differentiable. Then, for any stopping time  $T$  for which  $E[\int_0^T d'(t)f(X_t)dt]$  exists and is finite we have that

$$\begin{aligned} E[d(T)f(X_T)] - d(0)f(x) \\ = E[\int_0^T d'(t)f(X_t)dt] + E[\int_0^T d(t)Af(X_t)dt] \end{aligned}$$

We proved the following:

Theorem. Let  $f$  be concave increasing and  $c(x,z)$  be decreasing in the first argument. Let  $d$  be decreasing and concave. Let  $F$  be the class of stopping times for which  $E[\int_0^T d'(t)f(X_t)dt]$  exists and is finite. Define  $u(t) = d'(t)f(X_t) + d(t)Af(X_t)$ .

Let

$$T^* = \inf\{t: u(t) \leq 0\}$$

Then  $T^*$  is optimal in the sense that

$$E[d(T^*)f(X_{T^*})] = \sup_{T \in F} E[d(T)f(X_T)].$$

Theorem. Let  $f$  be concave increasing and  $c(x,z)$  be decreasing in the first argument. Let  $d(s) = e^{-\lambda s}$ ,  $\lambda \geq 0$ . Let  $F_1$  be the class of



stopping times for which  $E[\int_0^T e^{-\lambda t} f(X_t) dt]$  exists and is finite. Define  $A = \{x: Af(x) - \lambda f(x) \leq 0\}$ . Then the stopping time

$$T^* = \inf\{t: X_t \in A\}$$

is optimal in the sense that

$$E[e^{-\lambda T^*} f(X_{T^*})] = \sup_{T \in F_1} E[e^{-\lambda T} f(X_T)] .$$

### C) A PRACTICAL DEFINITION OF STABILITY FOR OPTIMAL STOPPING TIMES.

The results of this work represent progress on question 1(C) of the proposal. This question asked whether the two main types of stability are equivalent. While we do not provide a complete answer, the result which is obtained indicates that in a practical sense they can be considered to be equivalent.

Briefly, a problem  $(X, g)$  is considered to be center stable, here  $X$  is a process and  $g$  is a reward function defined on the state space of  $X$ , if a close to optimal solution to the problem  $(X, g)$  is close to optimal for any problem  $(X^1, g_1)$  which is sufficiently "close" to the problem  $(X, g)$ . The other broad class of stability considered views  $(X, g)$  as "stable" if a close to optimal solution to the problem  $(X^1, g_1)$  is close to optimal for  $(X, g)$  provided  $(X^1, g_1)$  is "close enough" to  $(X, g)$ .

We say that the problem  $(X, g)$  is weakly-payoff stable at  $x$  if for each  $\epsilon > 0$  there exists a positive integer  $n_0$  such that if  $X^1$  is "close enough" to  $X$  then  $\sup_{\tau \leq n_0} E_1^x g(X_\tau) \geq \pi_1 g(x) - \epsilon$  (here  $\pi_1 g$  is the smallest excessive

majorant of  $g$  with respect to  $X^1$ .) The problem  $(X, g)$  is said to be stable with respect to bounded times if for every  $\epsilon > 0$  and  $n_0 \in \{1, 2, \dots\}$  there is a  $\delta > 0$  and an  $\alpha > 0$  such that every  $\alpha$ -optimal time  $\tau \leq n_0$  for  $(X^1, g_1)$  is  $\epsilon$ -optimal for  $(X, g)$  whenever  $(X^1, g_1)$  is within  $\delta$  of  $(X, g)$ . The

concept of center stable with respect to bounded times is defined analogously.

The main result is the following:

THEOREM 1. The following are equivalent:

- a)  $(X, g)$  is center stable.
- b)  $(X, g)$  is center stable with respect to bounded times.
- c)  $(X, g)$  is weakly-payoff stable.
- d)  $(X, g)$  is weakly-payoff stable and stable with respect to bounded times.

This research intended to incorporate this result into a rewrite of his paper entitled, "Stability of Optimal Stopping Problems for Markov Processes".

#### D) STABILITY OF OPTIMAL REPLACEMENT PROBLEMS.

The results in this work represent progress on one of the main problems of the proposal, namely that of developing a stability theory for optimal replacement problems for wear processes which allows the modeling processes to have different lifetimes.

One considers a nonterminating wear process  $X$ , a fixed stopping time  $\Gamma$  with uniformly bounded expected lifetime, a family  $N(\Gamma)$  of stopping times which are bounded above by  $\Gamma$  and a family  $\bar{S}_X$  of related nonterminating processes for all of which  $\Gamma$  has uniformly bounded expected lifetime. A metric is placed on  $\bar{S}_X \times N(\Gamma)$  which reflects closeness of the processes in  $\bar{S}_X$  and of the stopping times in  $N(\Gamma)$ .

A problem  $(X, \zeta, g)$  is considered to be stable if a close to optimal replacement solution to the problem  $(X, \zeta, g)$  is close to optimal for any sufficient close problem  $(X', \zeta, g_1)$  and vice-versa.

If the lifetimes are fixed, we have stability. However, examples show that stability of replacement problems is quite sensitive to variation in the lifetimes. These results are currently being written up in a paper entitled, "Stability of Optimal Replacement Problems".

# E) AN ITERATION SCHEME FOR OBTAINING APPROXIMATE OPTIMAL REPLACEMENT POLICIES.

In this work we analyze the following iterative technique. Let  $b_1 = \frac{E^0 g(X_\zeta)}{E^0(\zeta)}$  where  $(X, \zeta, g)$  is a wear process with lifetime  $\zeta$ . Consider the problem of maximizing the criterion  $\psi(b_1, \tau) = b_1 E^0(\tau) - E^0 g(X_\tau)$  for  $\tau \leq \zeta$ . If we are interested in an  $\epsilon$ -optimal policy, then tolerances  $\alpha > 0$  and  $\beta > 0$  are determined in terms of  $X, g, \zeta$  and  $\epsilon$  so that, if a  $\beta$ -optimal policy  $\tau_1$  for the problem  $\psi(b_1, \tau)$  gives  $\psi(b_1, \tau_1) \leq \alpha$  then  $\zeta$  is already

$\epsilon$ -optimal, otherwise take  $b_2 = \frac{E^0 g(X_{\tau_1})}{E^0(\tau_1)}$  and repeat the steps on the criterion

$\psi(b_2, \tau) = b_2 E^0(\tau) - E^0 g(X_\tau)$ . Under very reasonable assumptions on  $g$  (it must be positive and bounded away from zero), it is shown that this iterative method will always supply an  $\epsilon$ -optimal replacement policy.

This method has several advantages over those being employed in the literature where enough is assumed about the process to associate a contraction mapping with the optimal replacement problem and/or the function  $g$  is so specialized as to permit the identification of a control policy by inspection of an associated formula employing the generator of the semigroup associated with  $X$ . This iterative method needs no such assumptions on  $g$  and works even when a true  $\epsilon$ -optimal policy does not exist. The lack of a  $\epsilon$ -optimal policy would frustrate all cases in the literature so far; especially the contraction mapping approach. These results are written up in the paper entitled, "An Iterative Scheme for Approximating Optimal Replacement Policies".

## 11. MEETINGS ATTENDED

1. The First Saudi Conference on Statistics and Applications,  
May 1 - May 5, 1983.
2. Denver meeting of the A.M.S.

3. April meeting of Seminar on Probability in Gainesville, Florida.
4. June meeting on Stochastic Failure Models in Charlotte, North Carolina  
(Gave a talk entitled, "Approximate Optimal Replacement Policies and Their Stability".)

